

De la poésie aux étoiles

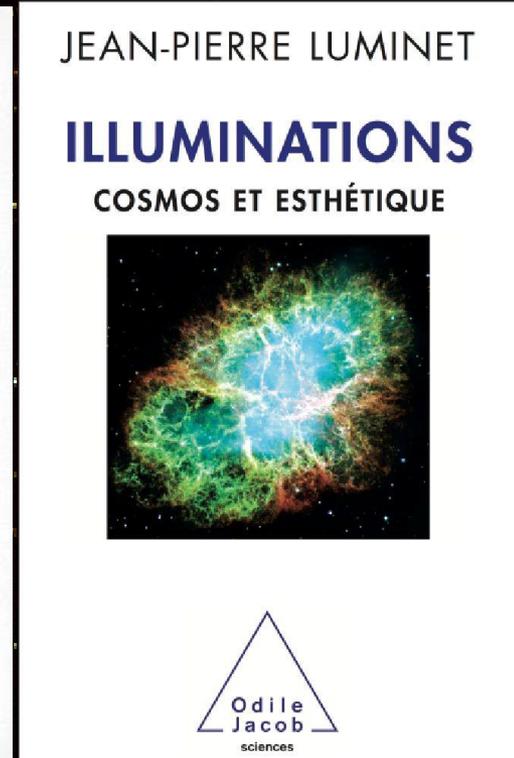
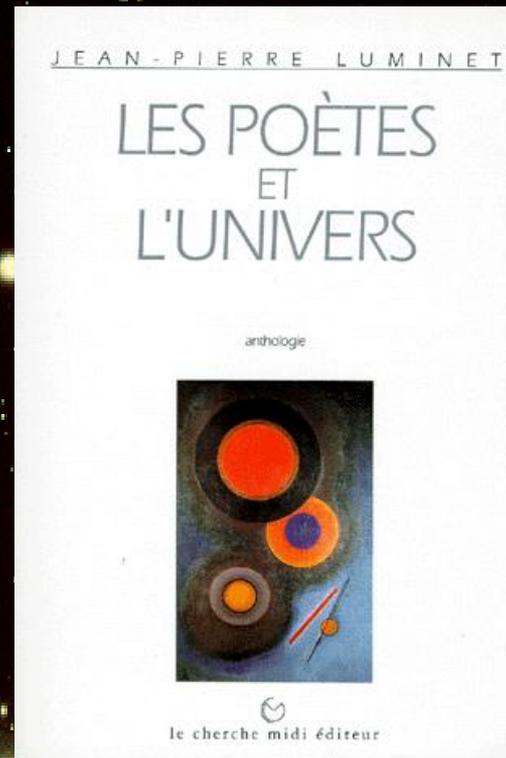
Le firmament

Ce fluide lumineux où
baignent les étoiles

Vous paraît sombre

Sa lumière est trop subtile
pour vos yeux

J.-P. L., Itinéraire céleste



Introducing the function $\Pi_{\beta\ell}(\xi) = R_{\beta\ell}(\xi) \sin^{1/2}(\xi)$ allows to solve the radial equation in terms of associated Legendre functions $P_{\nu}^{\mu}(\cos \xi)$.

For $K = +1$ (i.e. $\beta^2 = q^2 + 1$)^[4], the radial eigenfunctions are given by

$$R_{\beta\ell}(\chi) \propto \frac{1}{\sin^2 \chi} P_{-1/2+\beta}^{-1/2-\ell}(\cos \chi) \quad (\text{A.8})$$

with $\beta \geq \max(3, \ell + 1)$ being an integer^[4].

For $K = -1$ (i.e. $\beta^2 = q^2 - 1$) the radial eigenfunctions are given by

$$R_{\beta\ell}(\chi) \propto \frac{1}{\sinh^2 \chi} P_{-1/2+i\beta}^{-1/2-\ell}(\cosh \chi) \quad (\text{A.9})$$

and β can now take on any positive real value since there are no periodic boundary conditions to satisfy.

We use the normalisation condition

$$\int \mathcal{Y}_{\beta\ell m}^* \mathcal{Y}_{\beta'\ell' m'} \frac{r^2 dr d\Omega}{\sigma^3} = \delta(\beta - \beta') \delta_{\ell\ell'} \delta_{mm'}, \quad (\text{A.10})$$

so that the properly normalized functions take the following form

$$R_{\beta\ell}(\chi) = \begin{cases} \left(\frac{N_{\beta\ell}}{\sinh \chi} \right)^{1/2} P_{-1/2+i\beta}^{-1/2-\ell}(\cosh \chi) & K = -1 \\ (2\beta^2/\pi)^{1/2} j_{\ell}(\beta\chi) & K = 0 \\ \left(\frac{M_{\beta\ell}}{\sin \chi} \right)^{1/2} P_{-1/2+\beta}^{-1/2-\ell}(\cos \chi) & K = +1 \end{cases} \quad (\text{A.11})$$

with the two coefficients

$$N_{\beta\ell} \equiv \prod_{n=0}^{\ell} (\beta^2 + n^2) \quad M_{\beta\ell} \equiv \prod_{n=0}^{\ell} (\beta^2 - n^2). \quad (\text{A.12})$$

The radial eigenfunctions (A.11) differ from those determined by Abbott and Schaeffer [51] by an overall factor $(2\beta^2/\pi)^{-1/2}$ due to the fact that they used the normalisation

$$\int \mathcal{Y}_{\beta\ell m}^* \mathcal{Y}_{\beta'\ell' m'} \frac{r^2 dr d\Omega}{\sigma^3(r)} = \frac{\pi}{2\beta^2} \delta(\beta - \beta') \delta_{\ell\ell'} \delta_{mm'}. \quad (\text{A.13})$$

To finish, in the case of spherical space, the harmonic functions can be expressed in terms of the 4-dimensional coordinates [3] as

$$\mathcal{Y}_{k\ell m}(x) = \left[\frac{(2\ell + 1)(\ell - m)!}{4\pi(\ell + m)!} \right]^{1/2} \left(\frac{M_{\beta\ell}}{\sqrt{1 - x_0^2}} \right)^{1/2} P_{-1/2+\beta}^{-1/2-\ell}(x_0) P_{\ell}^m \left(\frac{x_2}{\sqrt{1 - x_0^2}} \right) \frac{x_1 + ix_2}{\sqrt{x_1^2 + x_2^2}}. \quad (\text{A.14})$$

Those expressions are of little value for numerical computation. There are two routes to compute numerically the eigenmodes. First, and as explained in Abbott and Schaeffer [51], one can use a recursive relation between $R_{\beta\ell}$, $R_{\beta, \ell-1}$ and $R_{\beta, \ell-2}$. Another efficient method [53] makes use of a WKB approximation. These two methods are complementary.

⁺ We recall that $\beta^2 = q^2 + K$.

^{*} It is well known that homogeneous harmonic polynomials of degree k on \mathbf{R}^4 restricted to S^3 are eigenmodes of the Laplacian with eigenvalues $k(k+2)$. It follows that $\beta = k-1$ is necessarily an integer.

2. Quarter turn space

The quarter turn space is similar to the half turn space, but with with a quarter turn corkscrew motion

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \mapsto \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ L_x/4 \end{pmatrix}. \quad (\text{35})$$

In particular this implies that $L_x = L_y$. The Invariance Lemma shows that when $(k_x, k_y) \neq (0, 0)$ the four-dimensional subspace $\{\Upsilon_{k_x, k_y, k_z}, \Upsilon_{k_y, -k_x, k_z}, \Upsilon_{-k_x, -k_y, k_z}, \Upsilon_{-k_y, k_x, k_z}\}$ is preserved as a set, while the mode $\Upsilon_{k_x, k_y, k_z} + i^{n_x} \Upsilon_{k_y, -k_x, k_z} + (-1)^{n_x} \Upsilon_{-k_x, -k_y, k_z} + (-i)^{n_x} \Upsilon_{-k_y, k_x, k_z}$ is fixed exactly. In the exceptional case that $(k_x, k_y) = (0, 0)$, the one-dimensional subspace $\{\Upsilon_{0,0,k_z}\}$ is preserved as a set, while $\Upsilon_{0,0,k_z}$ is fixed if and only if n_x is a multiple of 4. In summary, an orthonormal basis for the space of eigenmodes of the quarter turn space, $\Upsilon_{k_x, k_y, -k_z}^{[E_1]}$, is the union of

$$\begin{aligned} & \frac{1}{2} \left[\Upsilon_{2\pi(\frac{n_x}{L_x}, \frac{n_y}{L_y}, \frac{n_z}{L_z})} + i^{n_x} \Upsilon_{2\pi(\frac{n_x}{L_y}, -\frac{n_x}{L_x}, \frac{n_z}{L_z})} + i^{2n_x} \Upsilon_{2\pi(-\frac{n_x}{L_x}, -\frac{n_x}{L_y}, \frac{n_z}{L_z})} + i^{3n_x} \Upsilon_{2\pi(-\frac{n_x}{L_y}, \frac{n_x}{L_x}, \frac{n_z}{L_z})} \right] \\ & \quad \text{for } n_x \in Z^+, n_y \in Z^+ \cup \{0\}, n_z \in Z, \\ & \Upsilon_{2\pi(0,0, \frac{n_z}{L_z})} \quad \text{for } n_z \in 4Z. \end{aligned} \quad (\text{36})$$

As in the case of the half turn space, one can easily check that

$$\Upsilon_{k_x, k_y, k_z}^{[E_1]^*} = (-1)^{n_x} \Upsilon_{k_x, k_y, -k_z}^{[E_1]}, \quad (\text{37})$$

so that the analog of Eq. (25) is given by

1. when $k_x \neq 0$, \hat{e}_k is a complex random variable satisfying

$$\hat{e}_{k_x, k_y, k_z}^* = (-1)^{n_x} \hat{e}_{k_x, k_y, -k_z}, \quad (\text{38})$$

2. when $k_x = 0$, \hat{e}_k is a real random variable.

3. Third turn space

The third turn space is a three-fold quotient of a hexagonal 3-torus, not a rectangular one. To the generators (20) of the hexagonal 3-torus we add a one-third turn corkscrew motion

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} \mapsto \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ L_x/3 \end{pmatrix}. \quad (\text{39})$$

The eigenmodes Υ_k of the hexagonal 3-torus are already known from Eqn. (21) (and illustrated in Figure 5). Applying the Invariance Lemma to them with the additional generator (39) yields the eigenbasis, $\Upsilon_{k_x, k_y, k_z}^{[E_3]}$,

$$\begin{aligned} & \frac{1}{\sqrt{3}} \left[\Upsilon_k + \zeta^{n_x} \Upsilon_{kM} + \zeta^{2n_x} \Upsilon_{kM^2} \right] \quad \text{for } n_1 \in Z^+, n_2 \in Z^+ \cup \{0\}, n_3 \in Z, \\ & \Upsilon_{2\pi(0,0, \frac{n_z}{L_z})} \quad \text{for } n_3 \in 3Z, \end{aligned} \quad (\text{40})$$

where $\zeta = e^{2i\pi/3}$ is a cube root of unity and it is easily checked that

$$\begin{aligned} \mathbf{k} &= 2\pi \left(\frac{-n_2}{L}, \frac{2n_1 - n_2}{\sqrt{3}L}, \frac{n_3}{L_z} \right) \\ \mathbf{k}M &= 2\pi \left(\frac{n_1}{L}, \frac{2n_2 - n_1}{\sqrt{3}L}, \frac{n_3}{L_z} \right) \\ \mathbf{k}M^2 &= 2\pi \left(\frac{n_2 - n_1}{L}, \frac{-n_1 - n_2}{\sqrt{3}L}, \frac{n_3}{L_z} \right). \end{aligned} \quad (\text{41})$$

Cosmos



« Ordre »

« Harmonie »

« Beauté »

Raphaël :
Ecole
d'Athènes
1511



L'espace qui s'étend au-dessus du ciel n'a pas encore été chanté par aucun des poètes d'ici-bas, et ne sera jamais chanté dignement.

Platon (Phèdre)

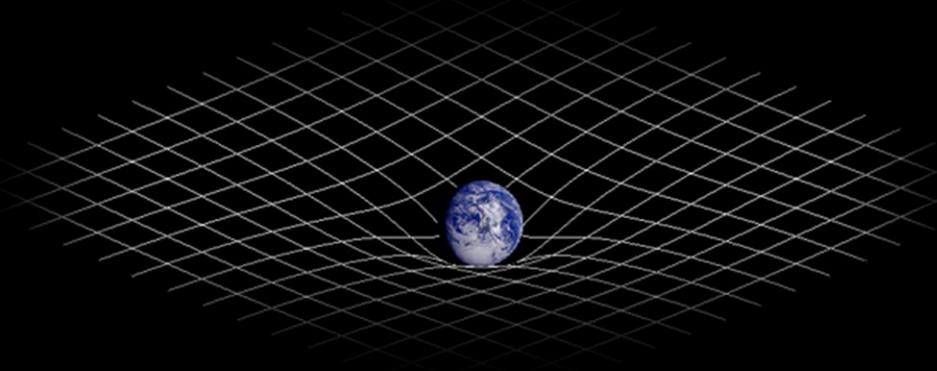
J'avais, à la faveur d'une grande sensibilité, compris la corrélation intime de la Poésie avec l'Univers, et pour qu'elle fût pure, conçu le dessein de la sortir du Rêve et du Hasard et de la juxtaposer à la conception de l'Univers.

Mallarmé



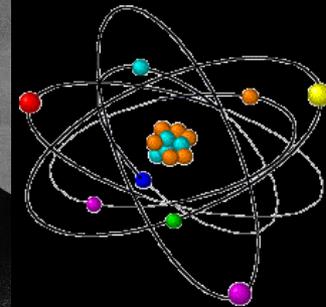
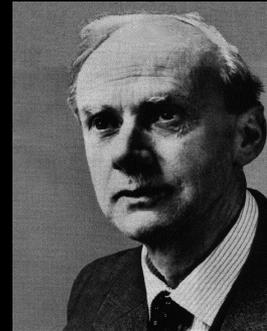


Relativité Générale



$$G_{\mu\nu} = k T_{\mu\nu}$$

Mécanique quantique



$$i\hbar \frac{\partial}{\partial t} \Psi = H \Psi$$

Equations d'Einstein : $G_{\mu\nu} = k T_{\mu\nu}$
(courbure = matière-énergie)

← Faible Gravité (Système Solaire)

Forte Gravité (Etoile dense)



Event
Horizon



**Très forte gravité:
Trou noir**

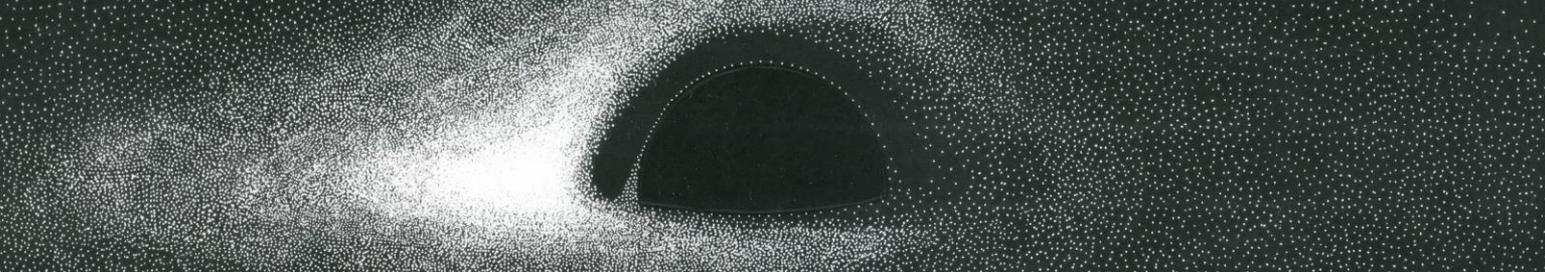
$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$$

The image features a dark, star-filled space background. In the center, there is a large, glowing orange sphere with a gradient from light orange at the top to dark orange at the bottom. The text "TROUGH NOIR !" is written across the middle of the sphere in a bold, yellow, italicized font with a white outline. The background is filled with numerous small, bright stars of various colors (white, yellow, blue) and several larger, faint galaxies or nebulae, including a prominent one in the upper right corner.



Image of a spherical black hole with thin accretion disk (J.-P. Luminet, 1979)

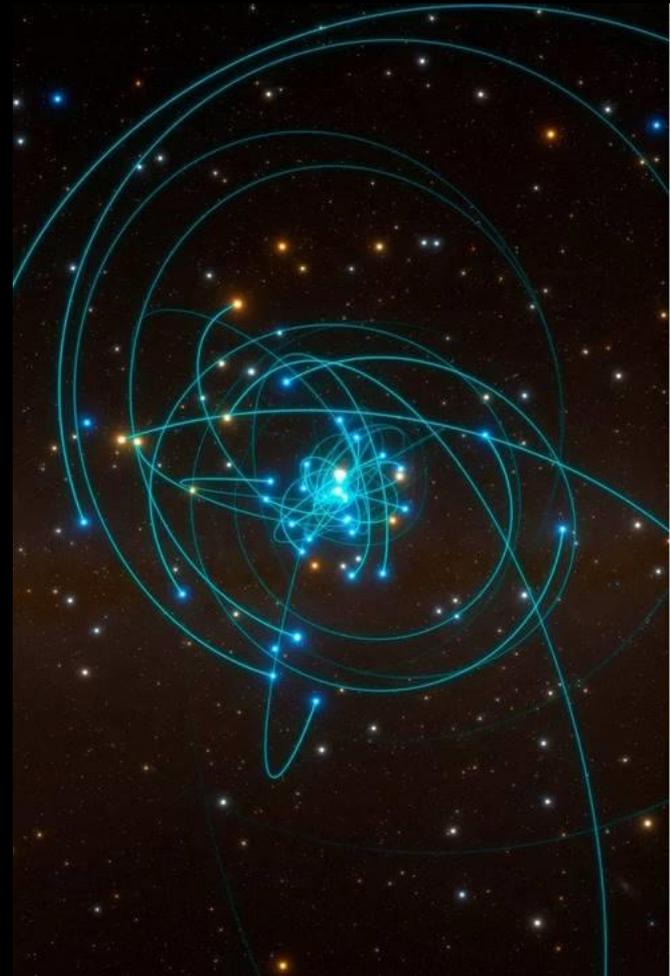
*En cherchant l'œil de Dieu, je n'ai vu qu'un orbite
Vaste, noir et sans fond, d'où la nuit qui l'habite
Rayonne sur le monde et s'épaissit toujours ;*



*Un arc-en-ciel étrange entoure ce puits sombre,
Seuil de l'ancien chaos dont le néant est l'ombre,
Spirale engloutissant les Mondes et les Jours !*

Gérard de Nerval, Chimères (1854)

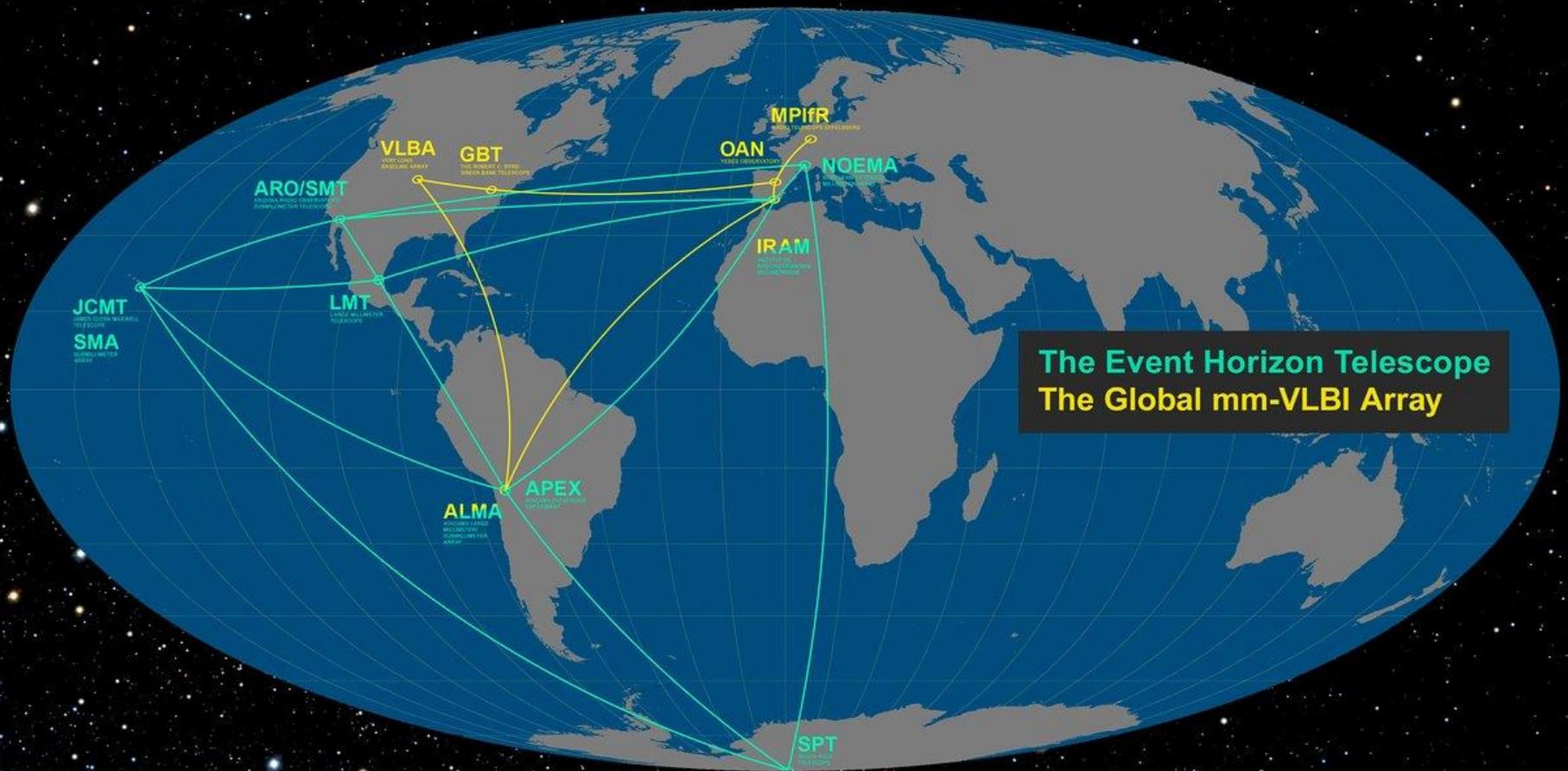
Le Trou noir Galactique



Masse \approx 4 000 000 masses solaires
Diamètre \approx 25 000 000 Km

Voir Sagittarius A* ?

Event Horizon Telescope !



Première image télescopique attendue en 2019 !

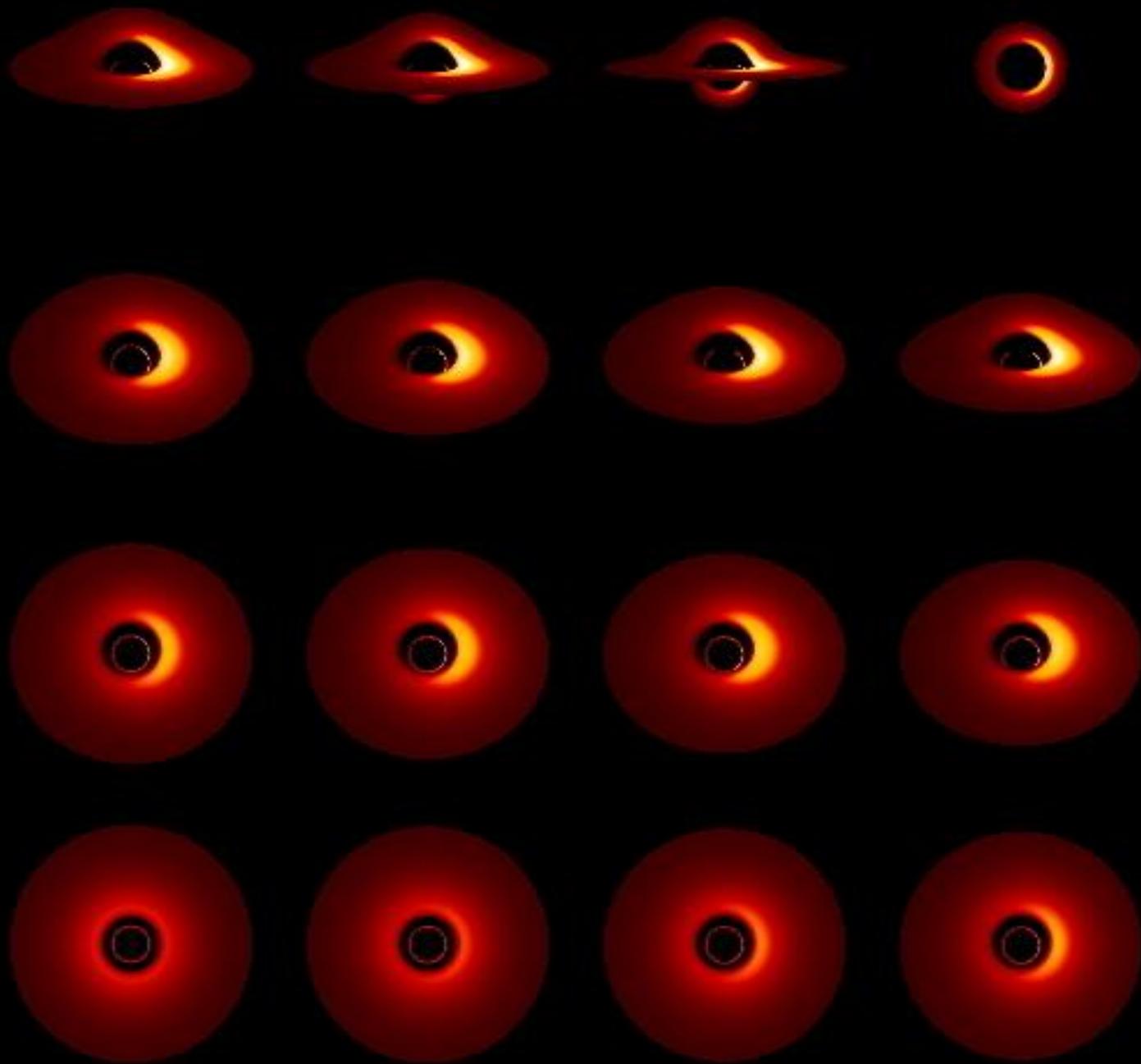
Reconstruction
d'image VLBI
de Sag A*
(2016)



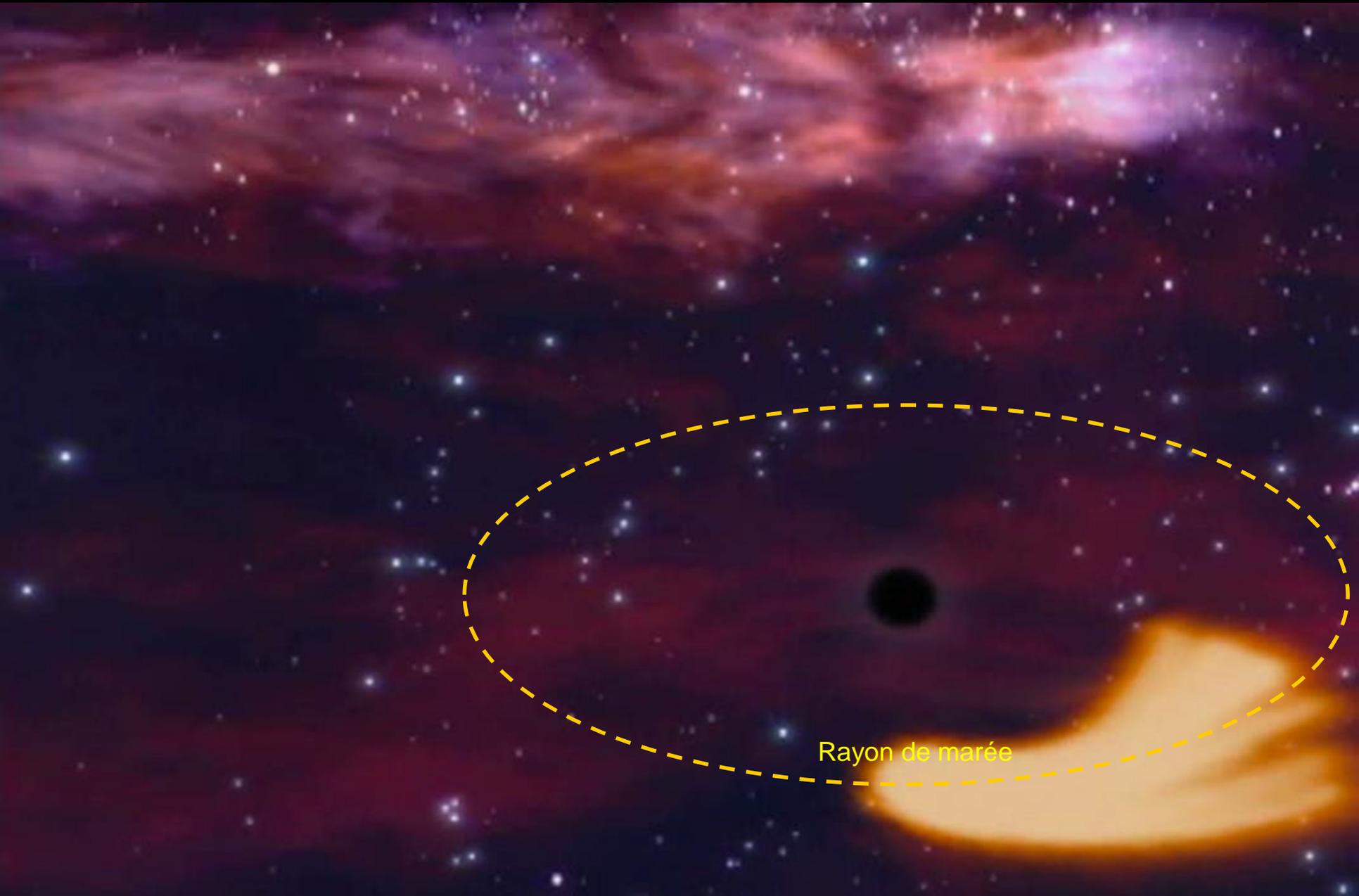
Image calculée
sous angle de
vue 30°
(JPL 1979)



JPL/
JA Marck
1989

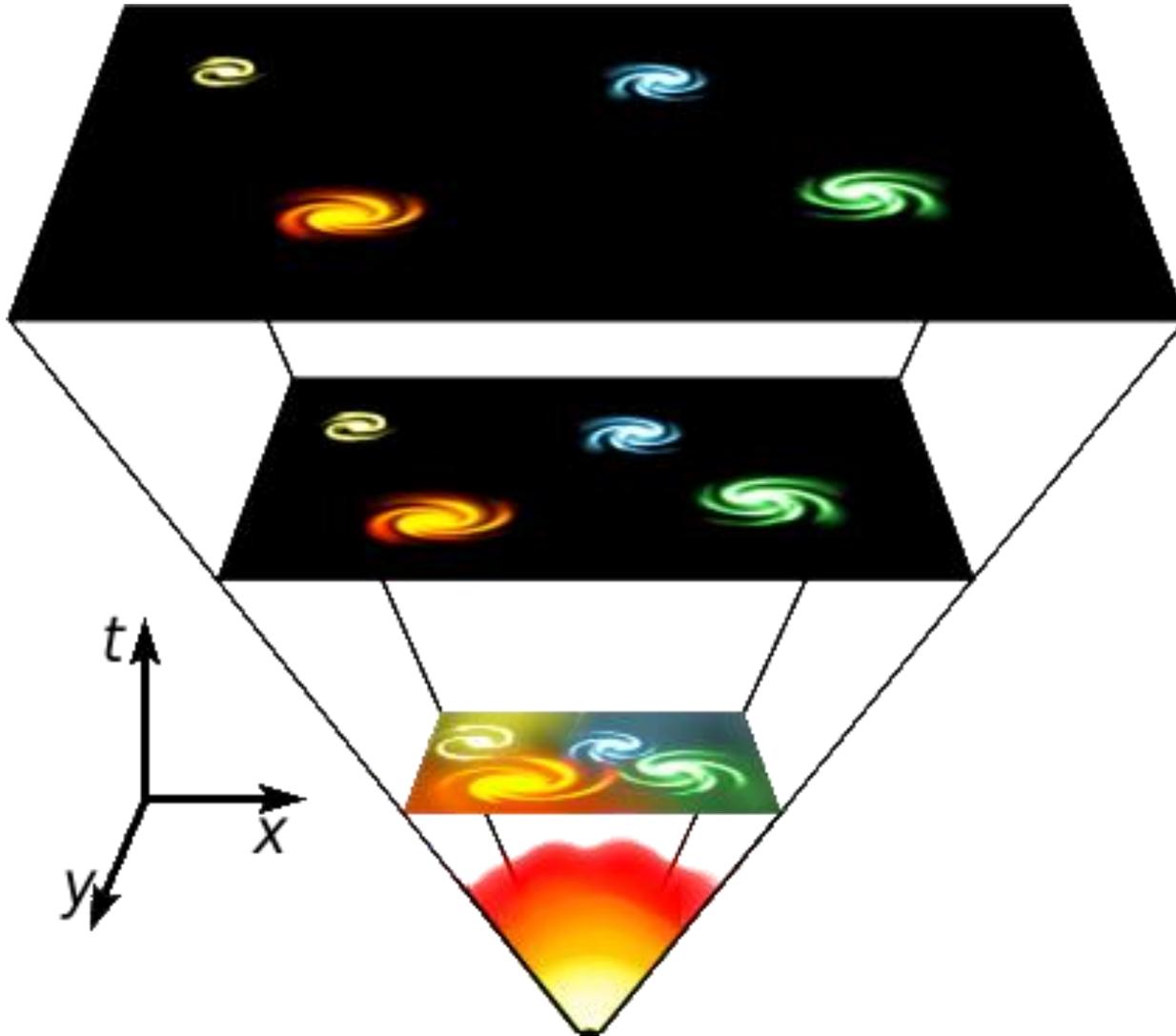


Les crêpes stellaires flambées



Rayon de marée

L'Univers est en expansion

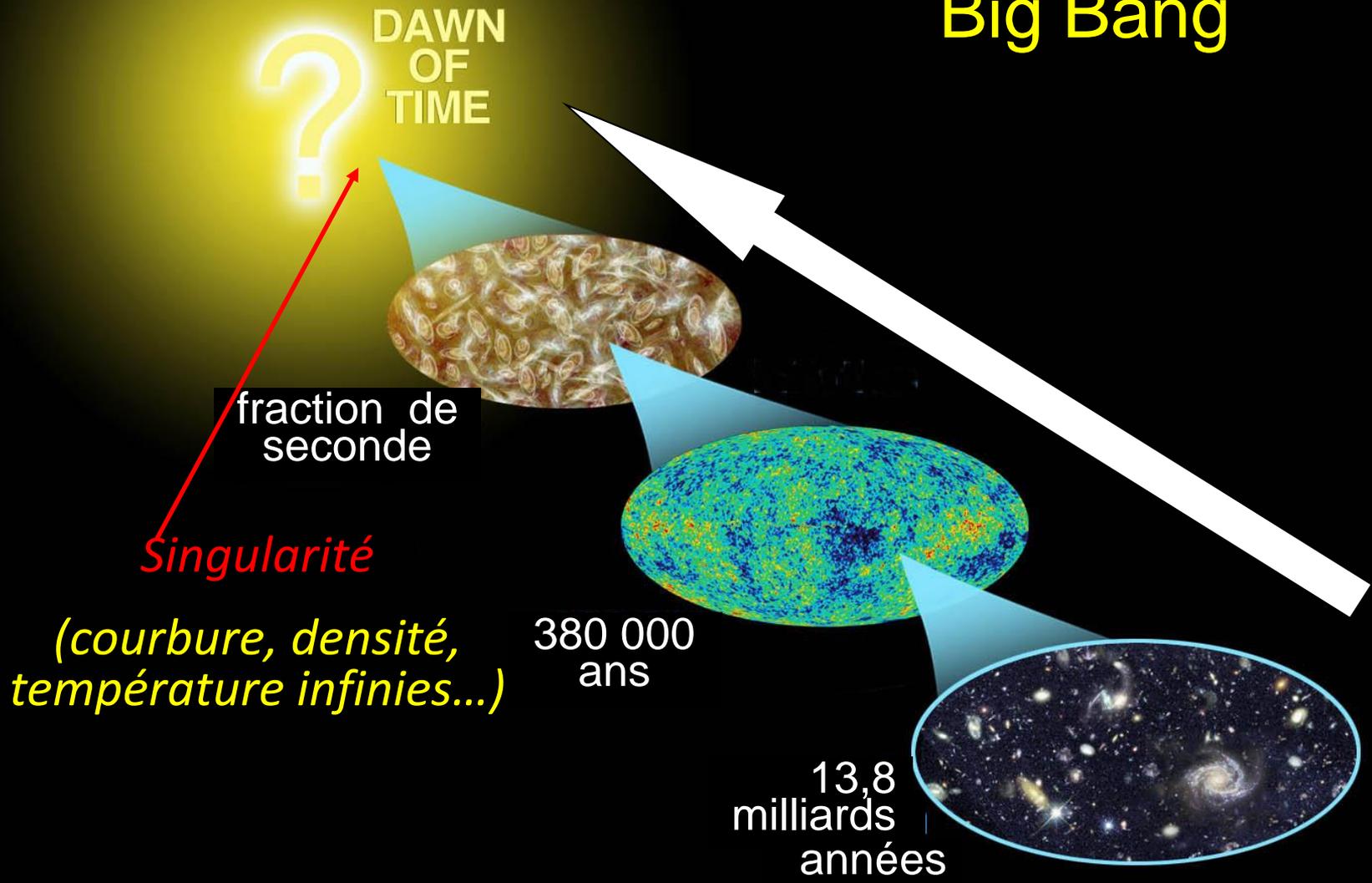


Big Bang

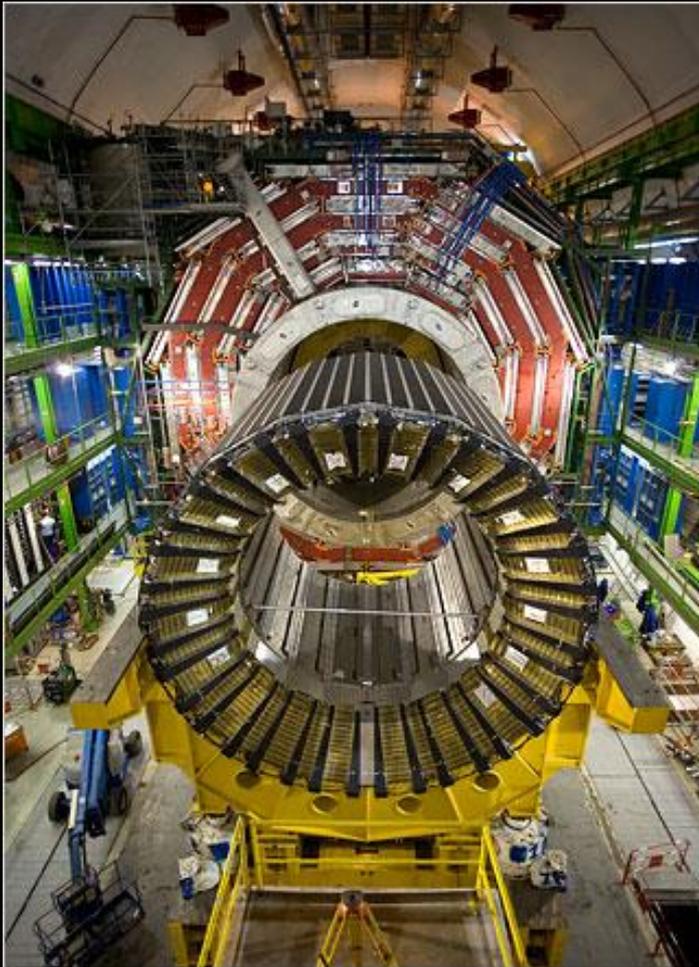
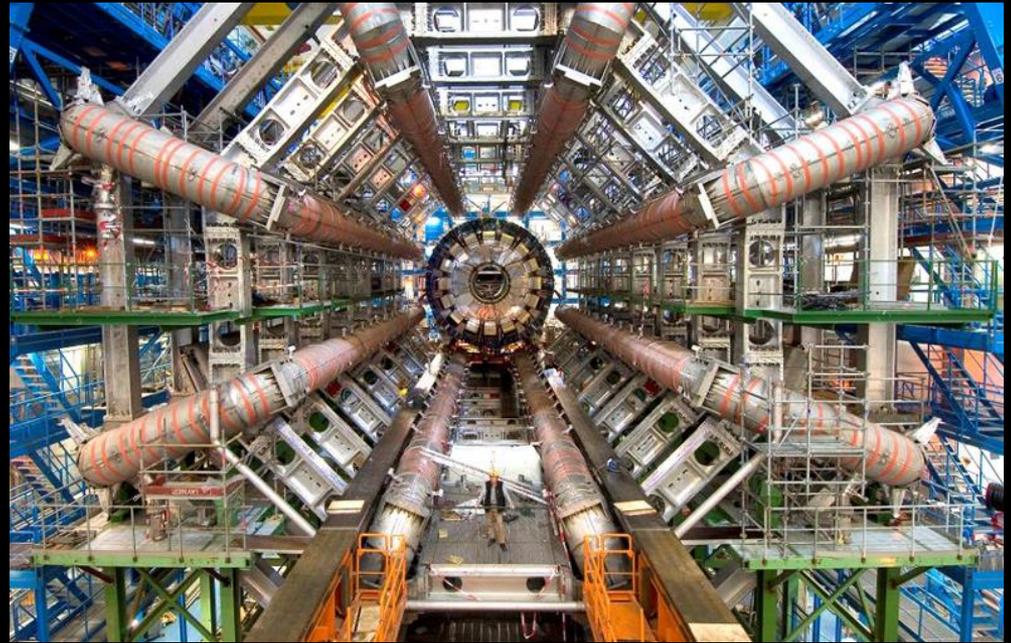


Univers
chiffonné

Modèle Standard du Big Bang



LHC (CERN)



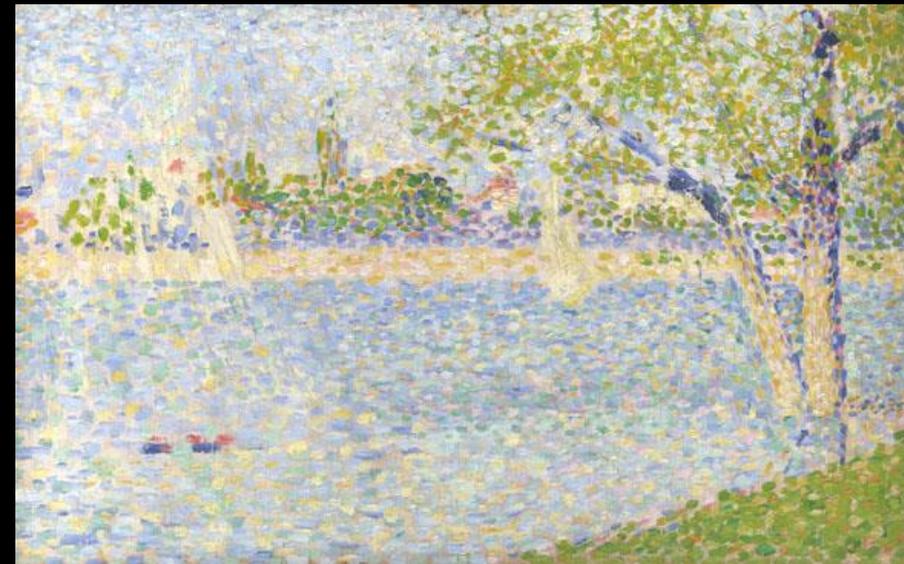
et au-delà ?

Relativité Générale



$$G_{\mu\nu} = k T_{\mu\nu}$$

Mécanique quantique

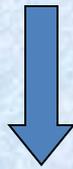


$$i\hbar \nabla^2 \Psi = H \Psi$$

↔
Gravité quantique

Solutions ?

Réduire la
géométrie aux
champs



Supercordes
Branes
M-theory

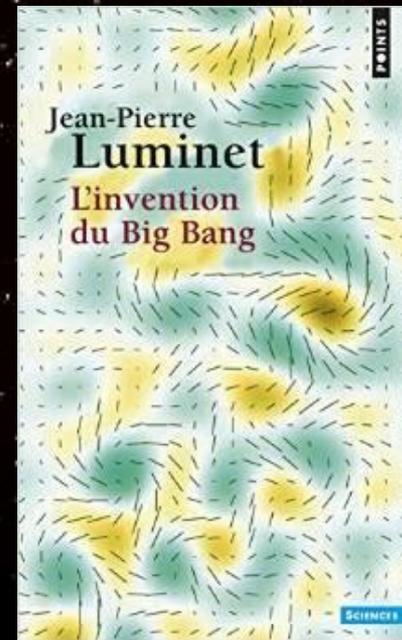
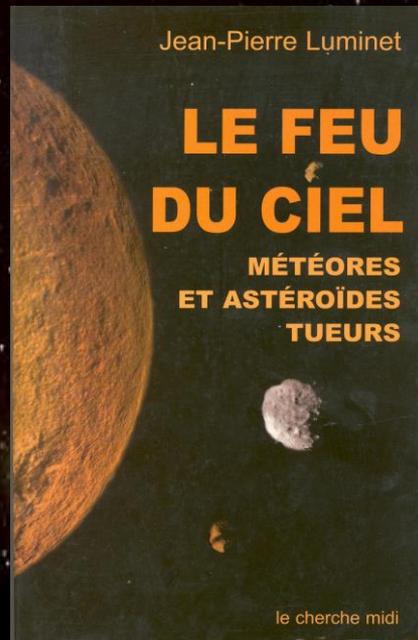
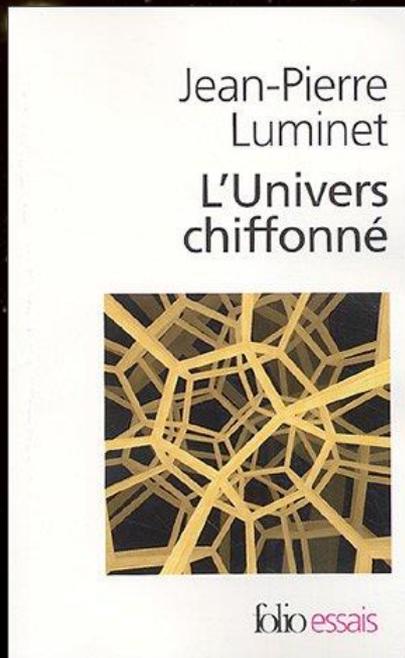
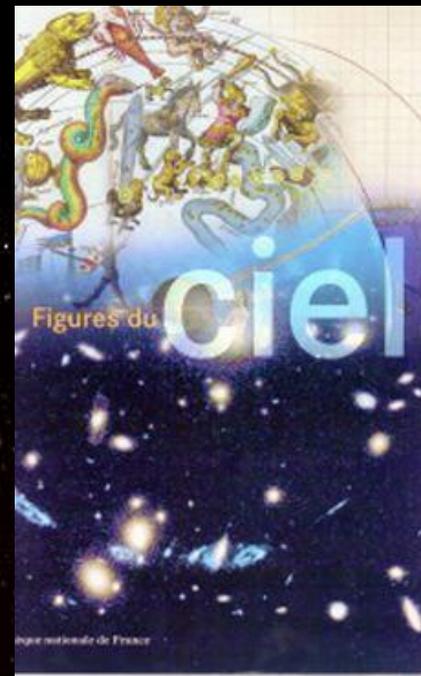
Réduire les
champs à la
géométrie

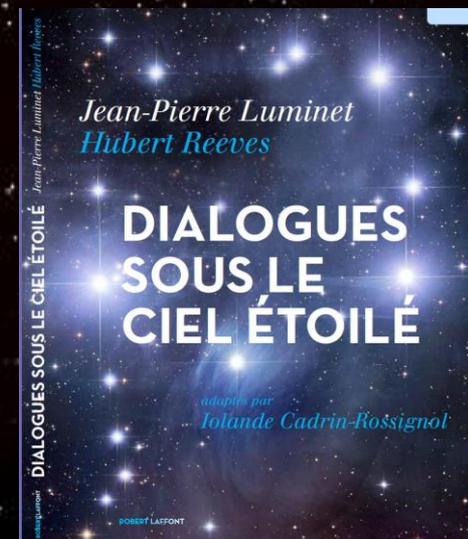
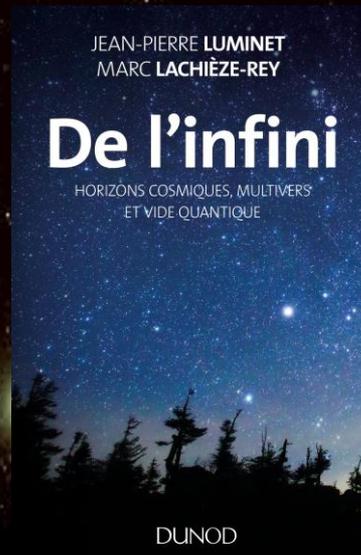
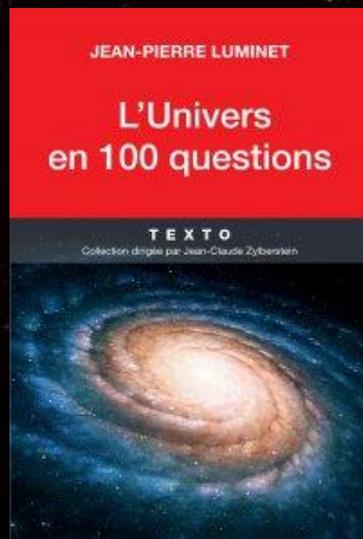
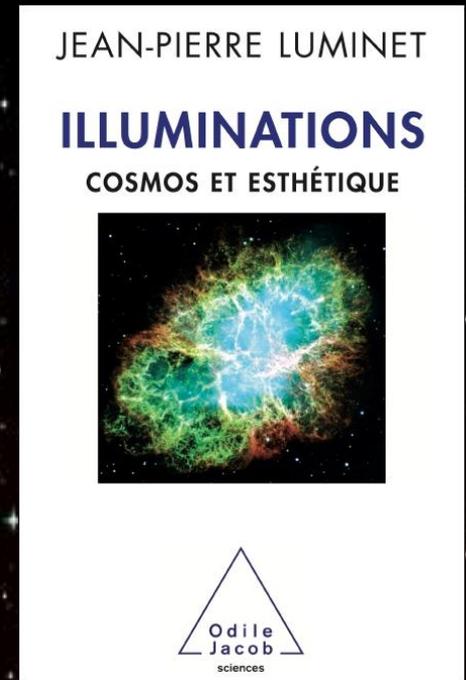
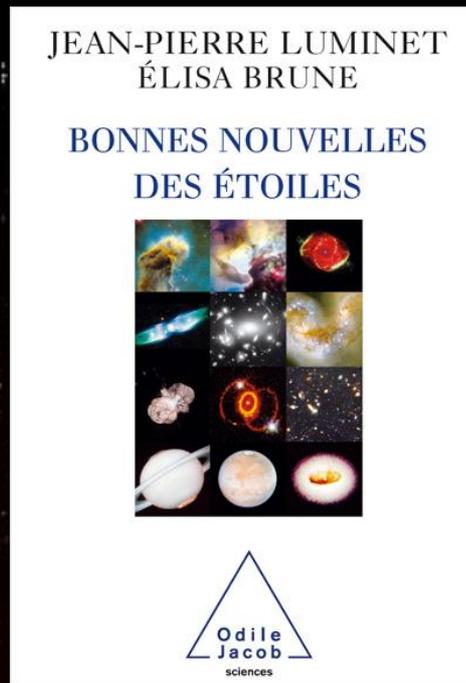
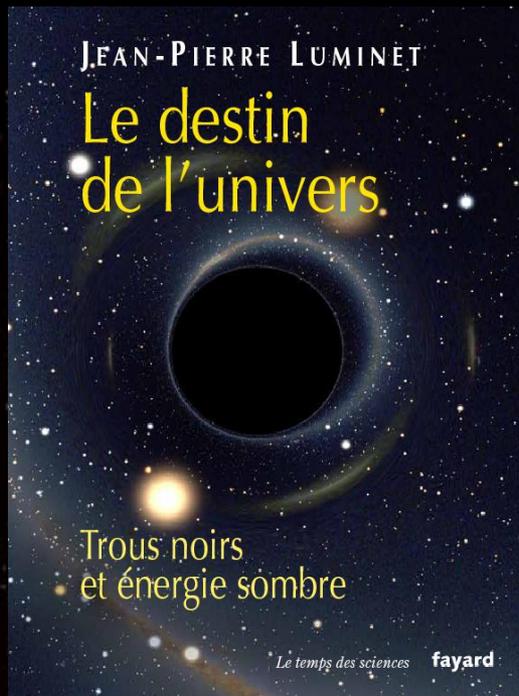


Gravité
quantique à
boucles

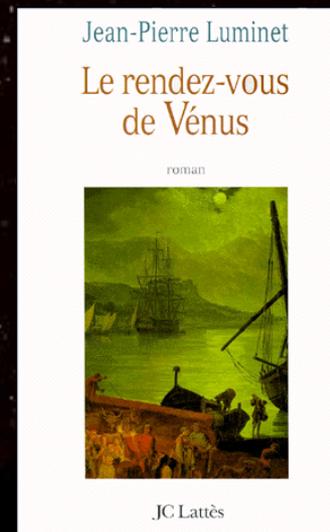
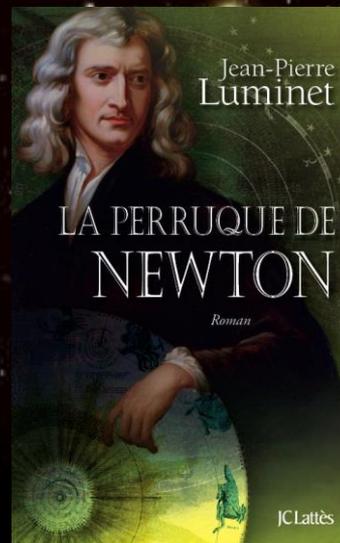
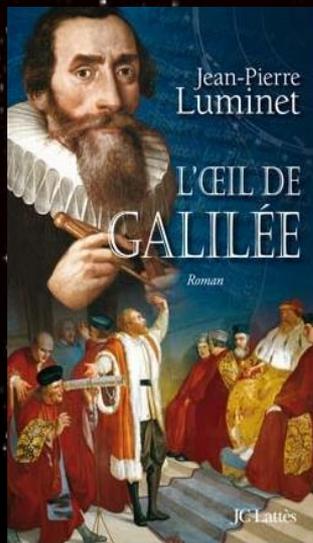
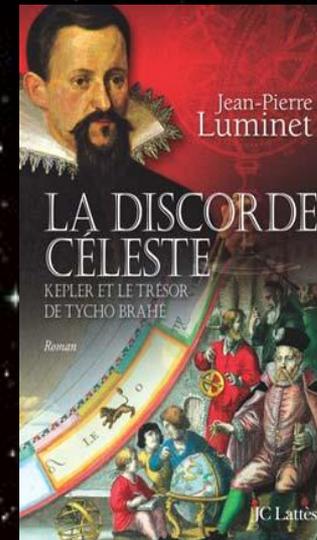
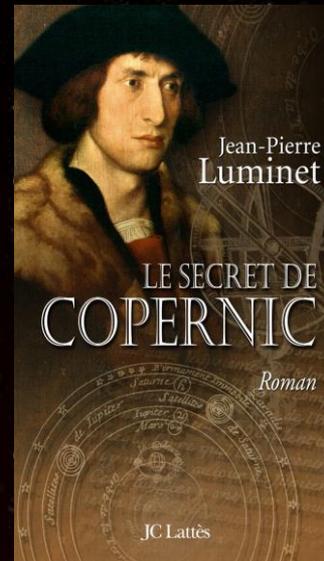
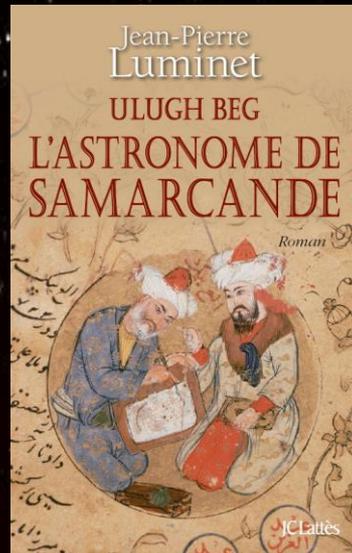
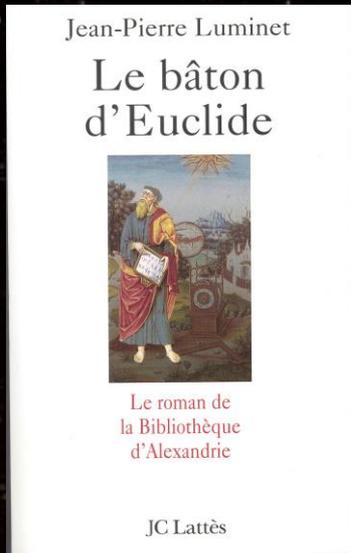
Melencolia ...







Les bâtisseurs du ciel



L'espace qui s'étend au-dessus du ciel n'a pas encore été chanté par aucun des poètes d'ici-bas, et ne sera jamais chanté dignement.

Platon

A la faveur d'une grande sensibilité, j'ai compris la corrélation intime de la Poésie avec l'Univers, et pour qu'elle soit pure, conçu le dessein de la sortir du Rêve et du Hasard et de la juxtaposer à la conception de l'Univers.

Mallarmé

JEAN-PIERRE LUMINET

LES POÈTES ET L'UNIVERS

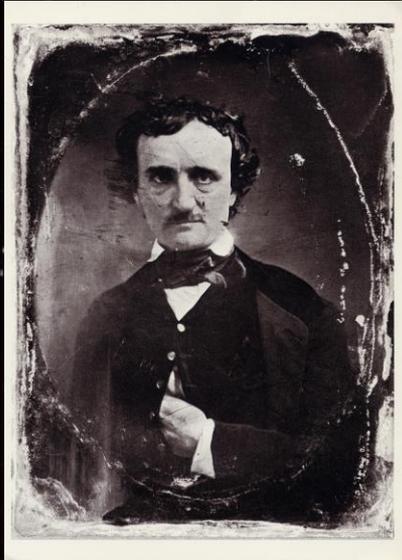
anthologie




le cherche midi éditeur

1996

Edgar Poe (1809-1849)



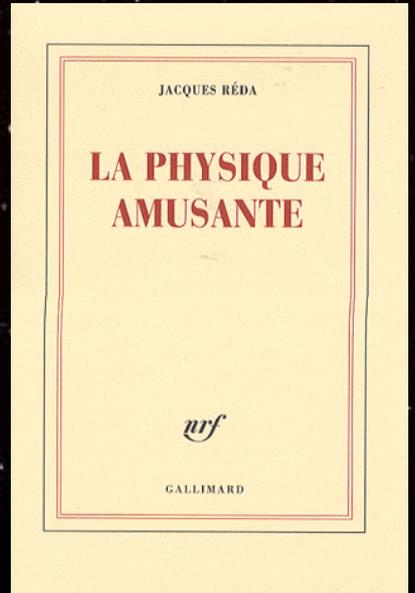
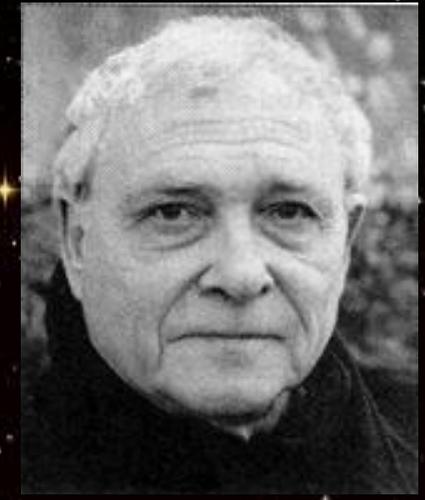
EURÉKA

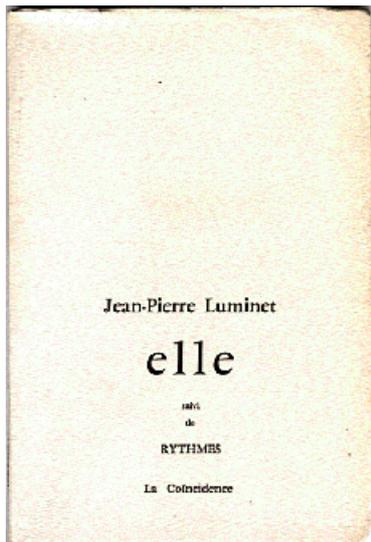
C'est avec une humilité non affectée, —
c'est même avec un sentiment d'effroi,
— que j'écris la phrase d'ouverture de
cet ouvrage; car de tous les sujets imaginables,
celui que j'offre au lecteur est le plus solennel,
le plus vaste, le plus difficile, le plus auguste.

Raymond Queneau (1903-1976)

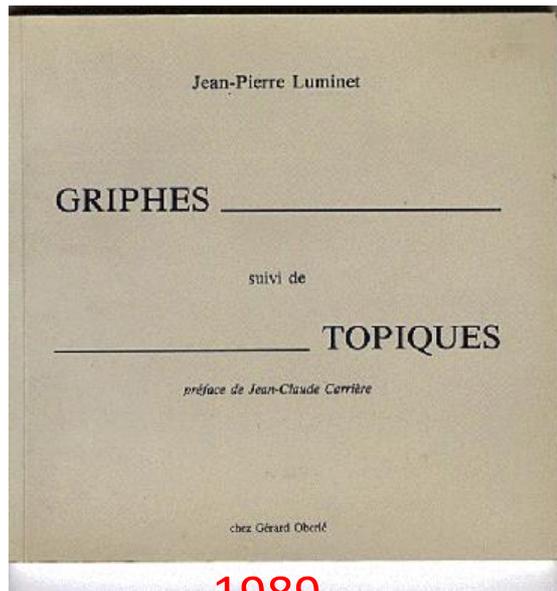


Jacques Réda (1929-)

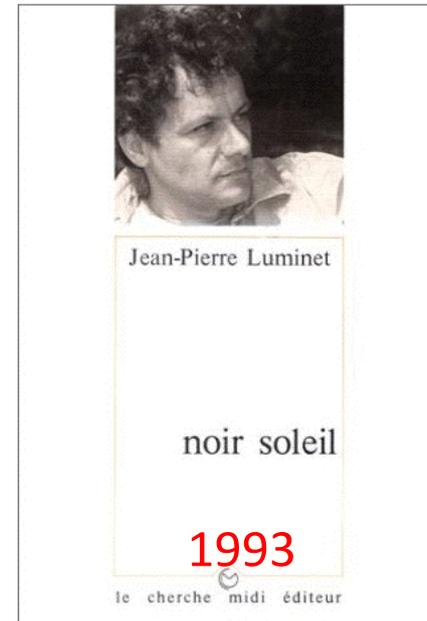




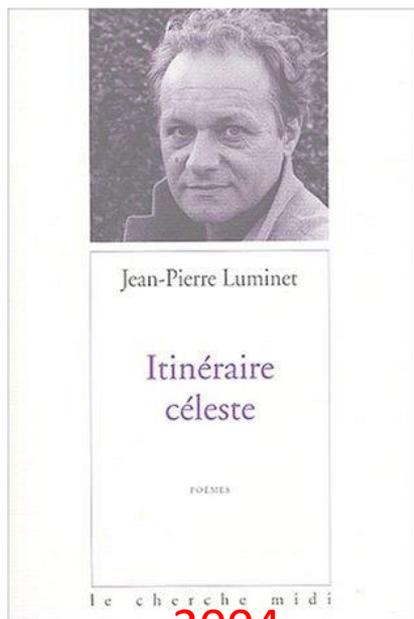
1980



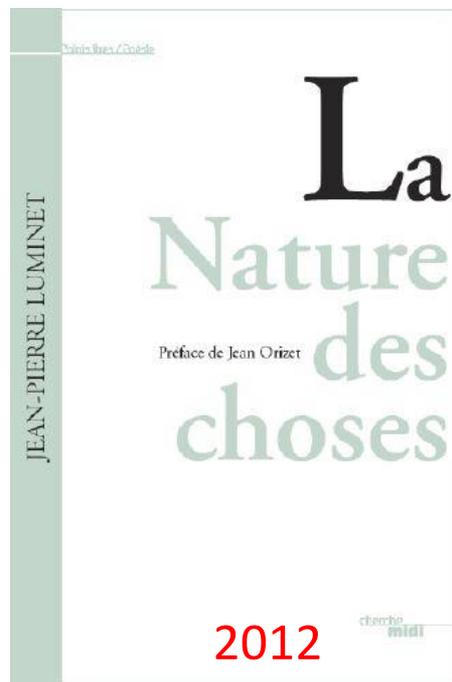
1989



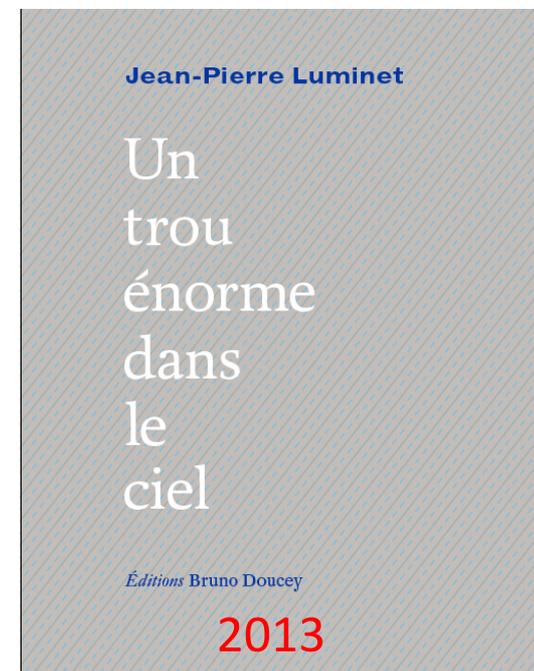
1993



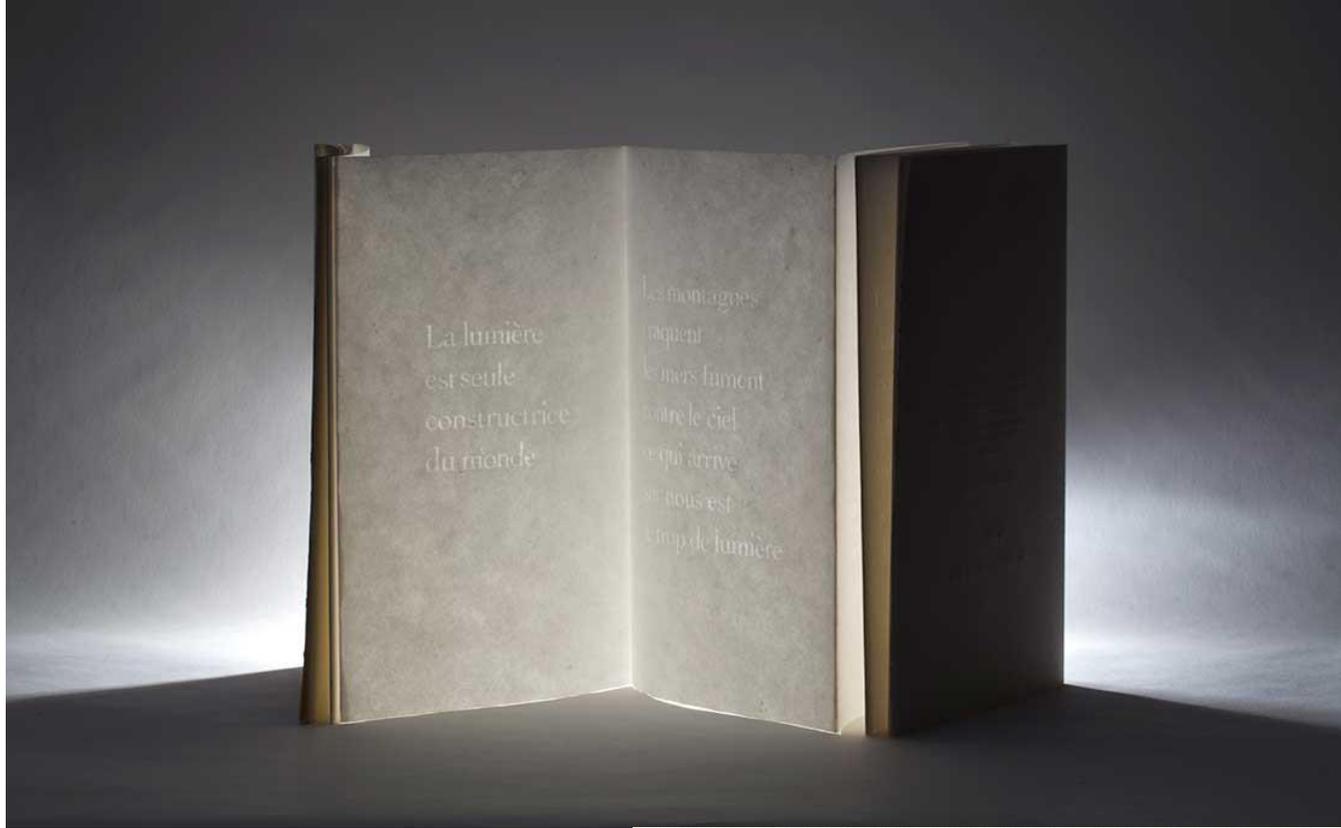
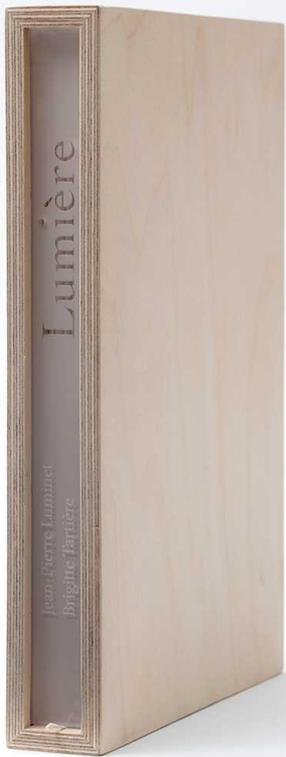
2004



2012



2013



Jean-Pierre Luminet
Cercles noirs
Couronnes lumineuses

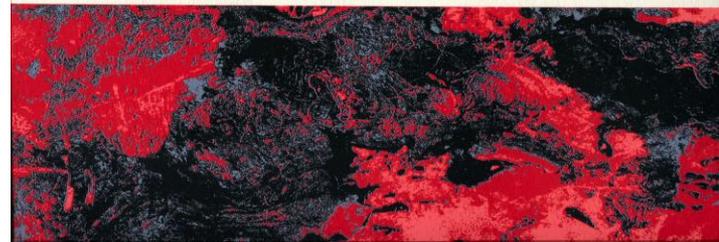


Vladimir Skoda

meridiens
collection Liber

Jean-Pierre Luminet
Bernard Louvel

Variations
sur la nature des choses





G. Grisey / J.-P. L. :
Le Noir de l'Étoile

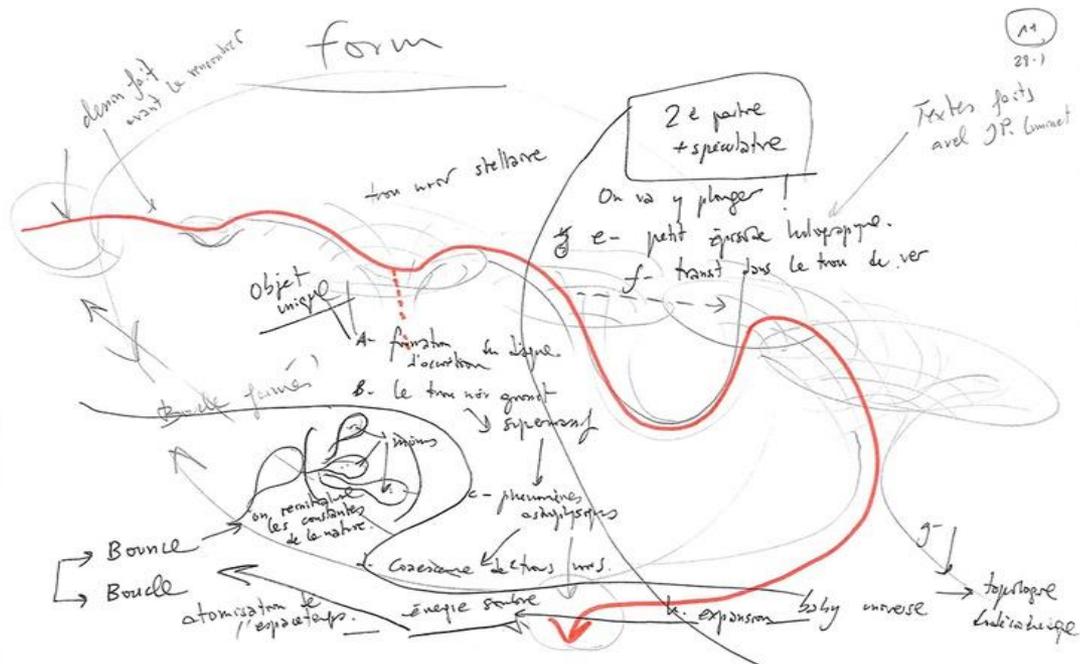
pour 6 percussionnistes placés autour du public,
bande magnétique et retransmission *in situ* de
signaux astronomiques
(1989-90)

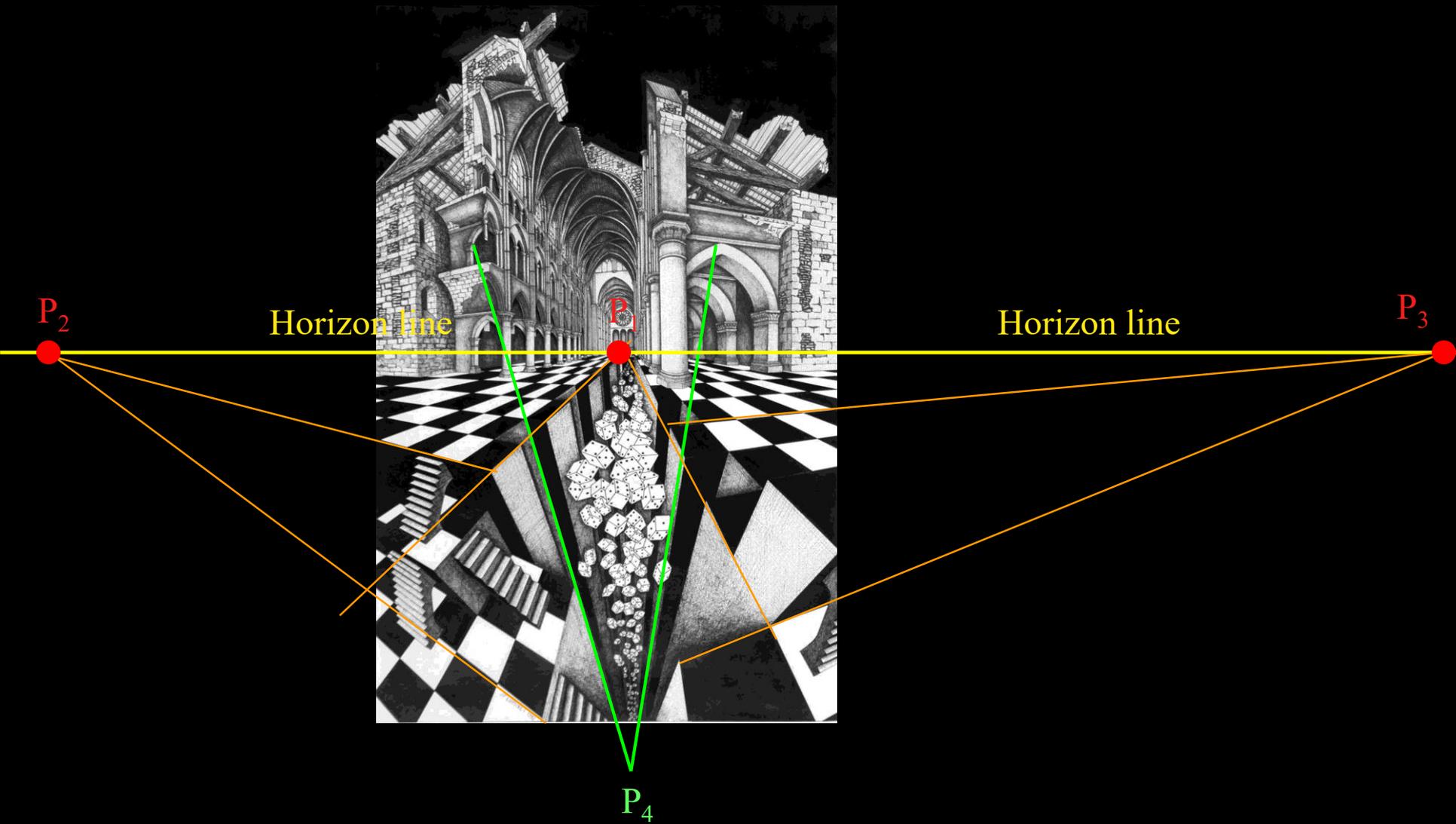
Texte de Présentation (JPL)
Premier mouvement
Pulsar Vela
Second mouvement
Pulsar 0329+54
Troisième mouvement

Hèctor Parra /JPL

Inscape (2018)

pour ensemble de 16
solistes, grand orchestre et
électronique







Cosmosaïques

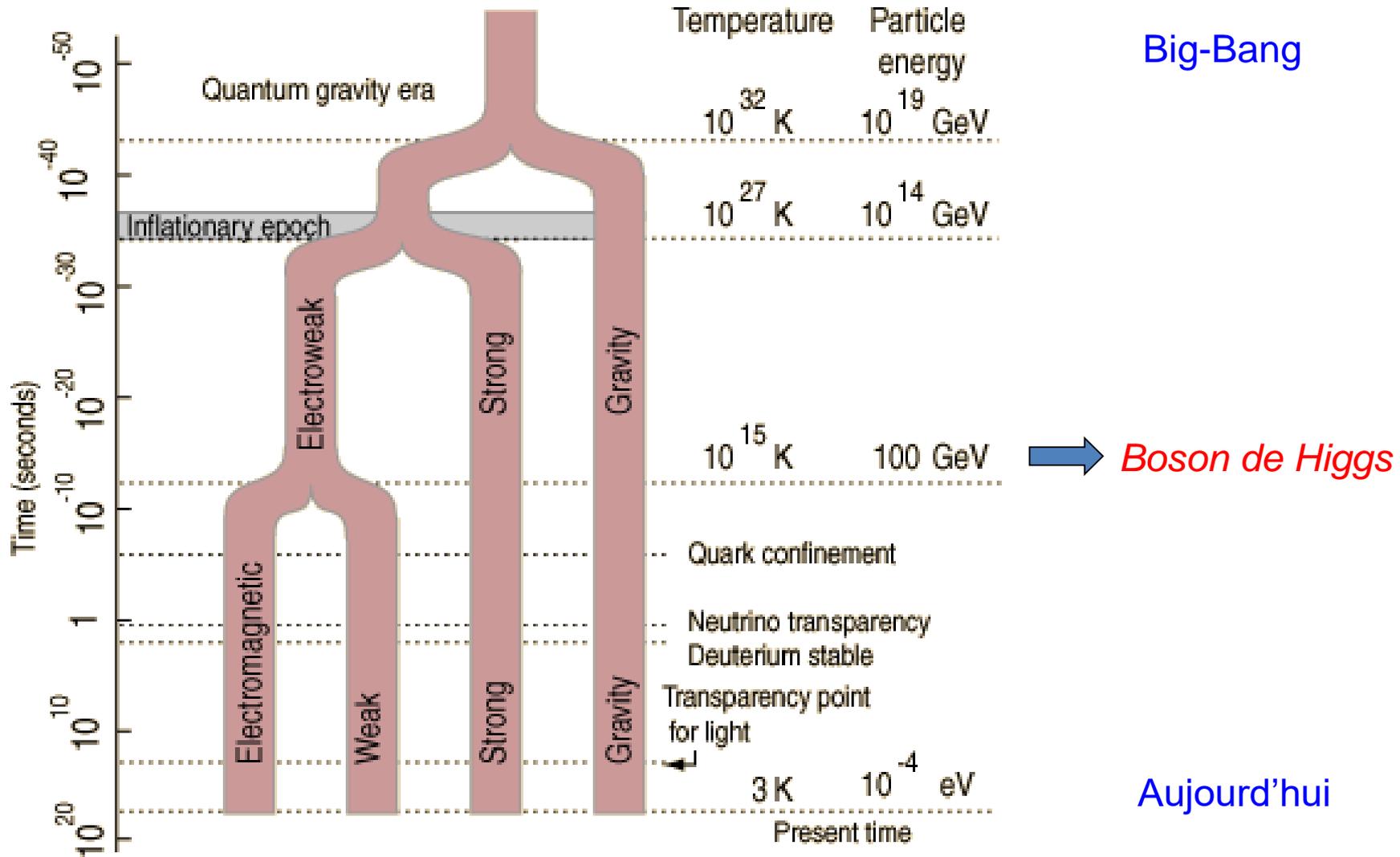
J.-P. Luminet, Collages (2005-2010)

Le sentiment de la symétrie est un instinct qui repose sur une confiance presque aveugle. C'est l'essence poétique de l'univers, de cet Univers qui, dans la perfection de sa symétrie, est simplement le plus sublime des poèmes

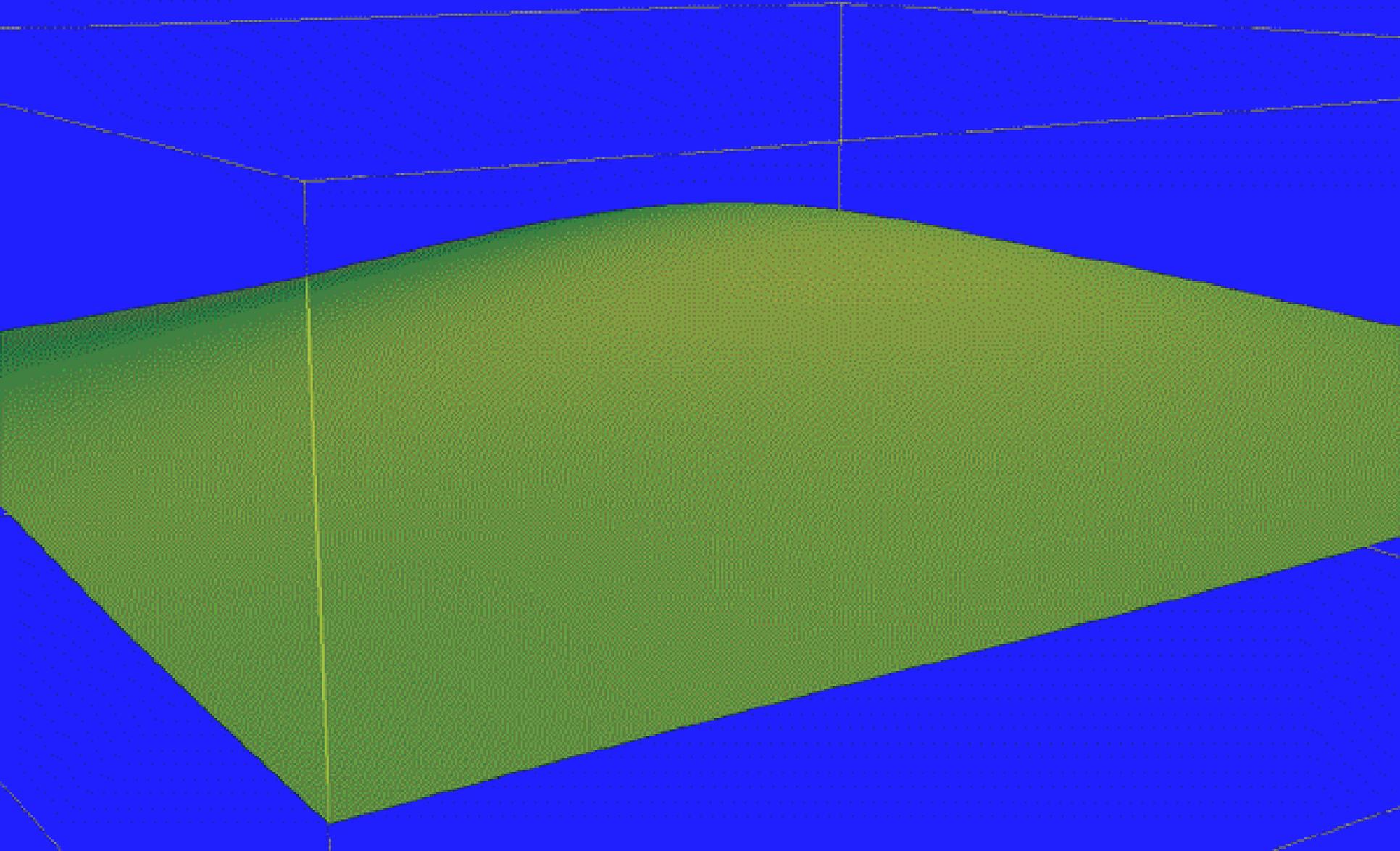
Edgar Poe (Eurêka)

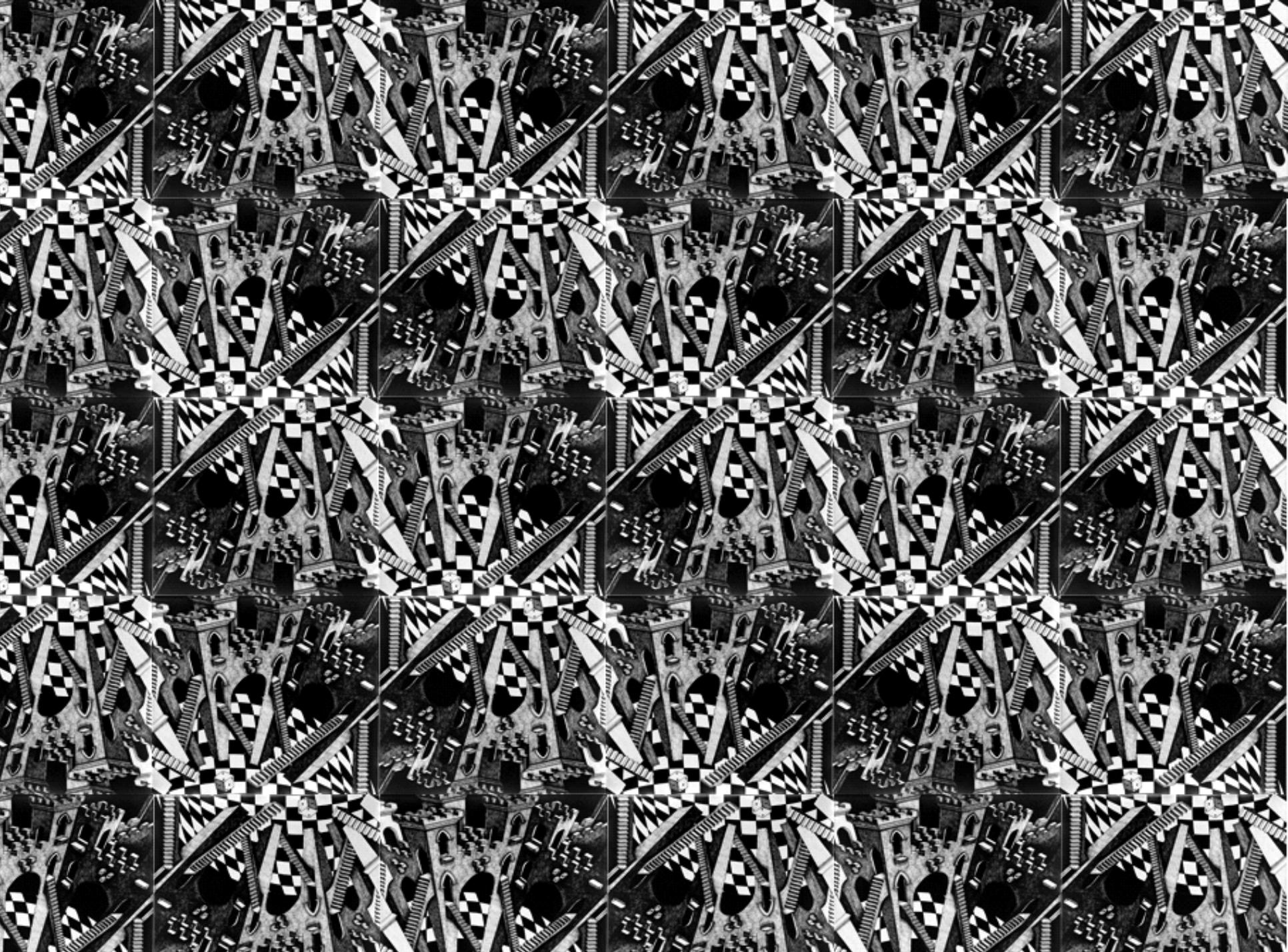
Aussi bien en Physique que dans les Arts, la
Brisure de Symétrie engendre la Dynamique

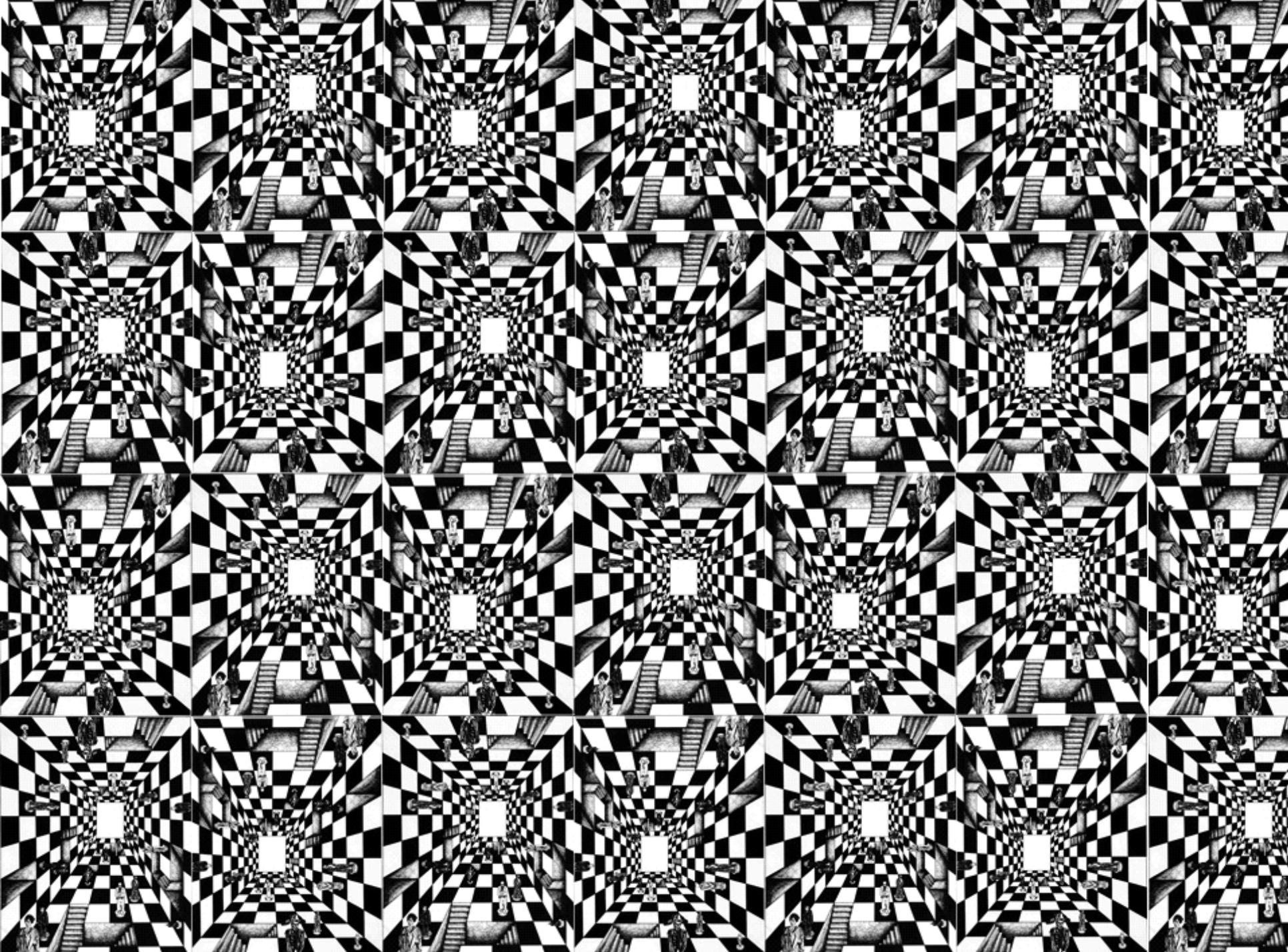
La **brisure de symétrie** en cosmologie engendre les **forces fondamentales** et les **particules**



Brisure de symétrie du vide quantique : un **multivers** ?







L' HOMME CHERCHE À SE FAÇONNER, DE FAÇON PLUS OU MOINS APPROPRIÉE, UNE IMAGE INTÉRIEURE DU MONDE, IMAGE SIMPLIFIÉE ET BIEN ORDONNÉE ; ET IL TENTE DE MAÎTRISER LE MONDE DE L' EXPÉRIENCE EN LUI SUBSTITUANT JUSQU' À UN CERTAIN POINT CETTE IMAGE. C' EST CE QUE FONT LE PEINTRE, LE POÈTE, LE PHILOSOPHE SPÉCULATIF ET LE CHERCHEUR SCIENTIFIQUE, CHACUN À SA FAÇON..

ALBERT EINSTEIN, 1931